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Analytics

Rethinking Retirement Risk

Increasingly, individuals in the U.S. are expected to plan and prepare for their own retirement through self-directed defined contribution (DC) plans. This is extremely complex: They must incorporate uncertainty about future asset returns and inflation, how these interact with their non-retirement assets, uncertainty over their own life expectancies and longevity risk, and all the while satisfy their income needs during retirement. Unfortunately, most financial advice and analysis regarding self-directed retirement plans focus solely on the plan's assets. As a result, they often mischaracterize both the magnitude and the fundamental nature of the risks associated with retirement planning for the typical household.

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An analytically robust framework for retirement plan design may help savers evaluate and manage the risks to their retirement portfolios. This framework incorporates salient features of a household's non-retirement assets, retirement benefits, future income trajectories and savings rates. There are three key conclusions:

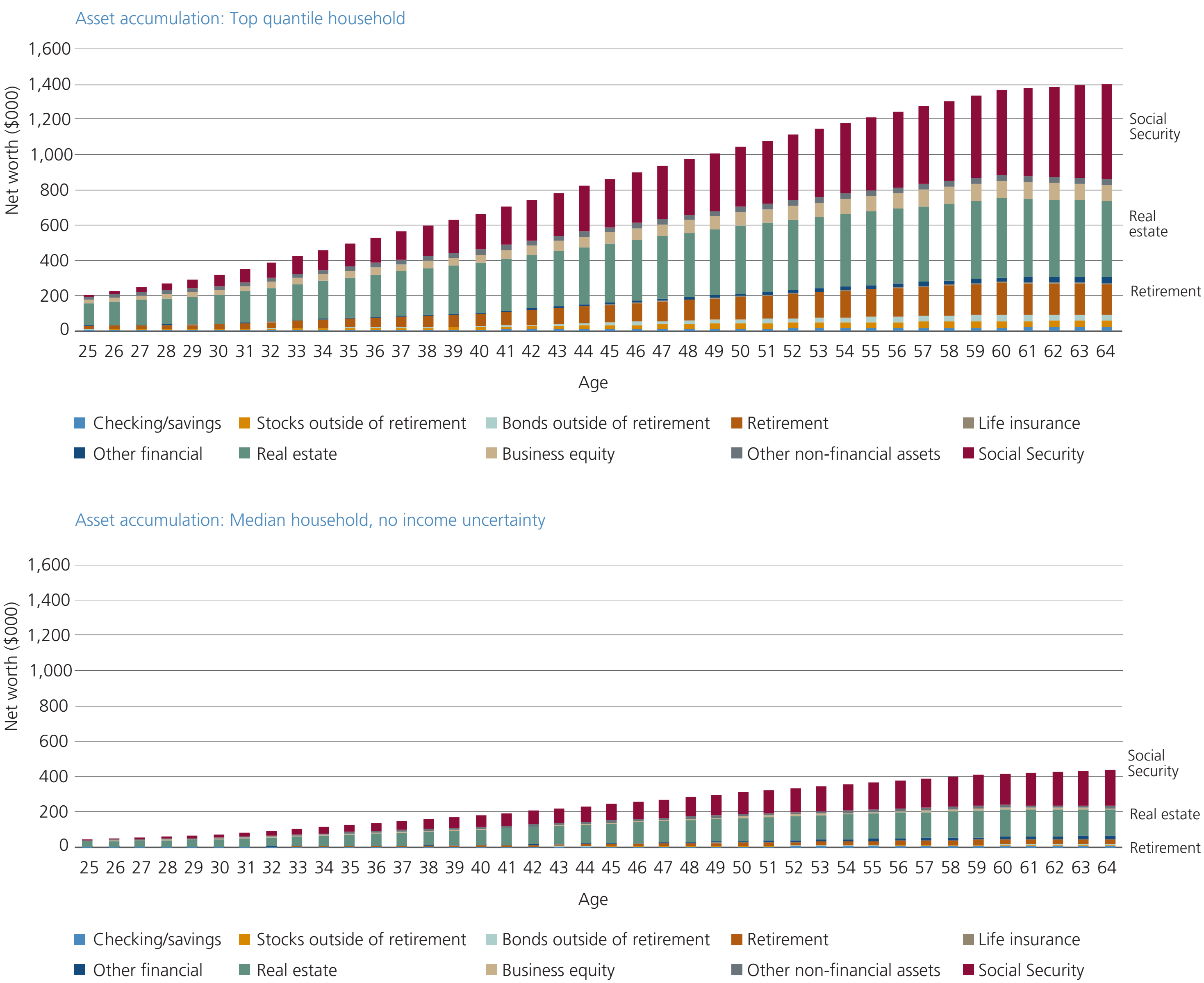
1. Glide path allocations that "de-risk" as investors near retirement are a natural result of future labor income and the corresponding future retirement savings.
2. Modeling the relationship between retirement liabilities and household assets is critical to understand the true risks of any glide path. The attainable level of income throughout retirement is closely related to changes in the real interest rate.
3. Accounting for the non-DC components of wealth and retirement income, such as accumulated home equity and Social Security benefits, is very important. Their presence on the household balance sheet has a meaningful impact on the design of the most efficient glide path.

Defining the problem

Household wealth accumulation over individuals' working lifetime is predominantly determined by their career earnings, savings rates and the allocation of savings across various financial assets and real estate.

While our analysis ultimately is focused on solving for the optimal asset allocation of assets within DC retirement plans, it is first necessary to specify realistic assumptions for household income and savings rates and to incorporate plausible assumptions for the allocation of non-DC savings. Our

FIGURE 1: ASSET BALANCE ACCUMULATION[1]



Source: Survey of Consumer Finances (2010), PIMCO.

assumptions are based on data from the Federal Reserve's Survey of Consumer Finances, the Bureau of Labor Statistics and Vanguard.

Historically, households have reported their retirement as the most important motive for saving in the U.S. Despite this statistic, retirement accounts constitute only a small portion of U.S. households' combined assets. Figure 1 shows the median asset balances for high-income and median-income households from the 2010 Survey of Consumer Finances. According to the survey, "retirement" assets (including both defined contribution and defined benefit amounts) account for only about 10% of total assets at the most common retirement age of 65. The bulk of retirement wealth is made up of housing wealth (30%–35%) and the present value of households' Social Security benefits (about 40%–50%).

Social Security is designed to guarantee a minimum level of consumption in retirement. The benefit for each individual is based on marital status and his or her labor earnings history. Benefits are progressive in the sense that households with lower lifetime income receive more benefits for each dollar paid in taxes than higher-income households. Recent estimates suggest that the real income replacement rate – defined as the fraction of labor income immediately preceding retirement that is provided by Social Security – ranges from 18% up to 85%, depending on the level of lifetime income (median-income households receive estimated replacement rates of roughly 40%). In addition, Social Security benefits are explicitly inflation-indexed, and thus can be thought of as a real annuity.

The significance of this program to the typical household merits its inclusion in any analysis of retirement programs within the U.S. Barring large changes in labor income during the working years, the accrued replacement rate in retirement will be relatively predictable during an individual's working life. In our analysis, we calculate Social Security benefits based on the labor income profile of the typical household in the 60th–80th percentile of the income distribution from the Survey of Consumer Finances. We focus on this portion of the distribution due to the relative importance of retirement savings: For households of median income and below, the preponderance of retirement income comes from Social Security, with relatively small private retirement accounts. For the highest-income households, there are typically other large sources of wealth that may be more readily tapped for retirement income. This results in an estimated income replacement rate from Social Security of 38%.

The importance of Social Security notwithstanding, housing wealth is another seemingly important component of total wealth at retirement. The empirical evidence suggests that retired individuals are reluctant or unable to finance their retirement with the equity they have accumulated in their homes. They typically stay in their home for as long as possible, generally only downsizing relatively late into retirement after significant health expenses or the death of a spouse. There are increasingly many avenues, such as reverse mortgages, that allow households to tap into their housing wealth during retirement without relocating. For the purposes of our analysis, we assume that households will be able to use of some of their accumulated housing wealth to finance their retirement, but we apply a haircut of 50% to reflect the reluctance of households to monetize housing, the difficulty involved in doing so and future housing consumption needs throughout retirement.

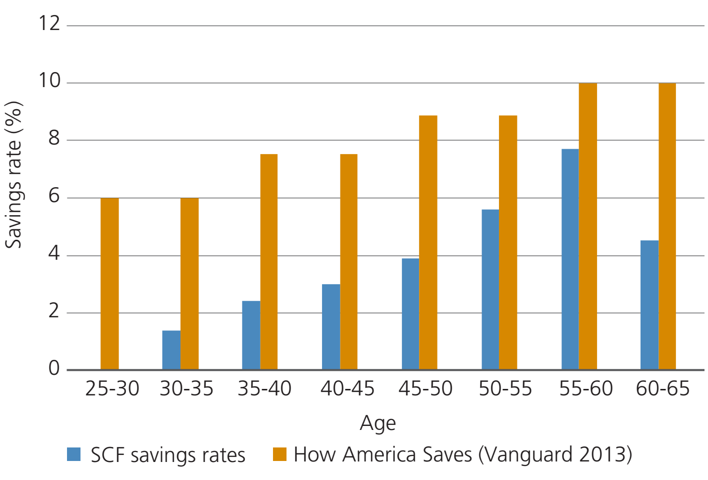
Lifecycle income and savings profiles

One area where there might be meaningful differences between future behavior and historical experience is DC plan contribution and savings rates. Over the last decade, default enrollment into DC plans has become much more common throughout the U.S. This has broadly increased participation rates, though with low default savings rates (about half of plans with automatic enrollment set the default contribution rate at 3%). As a response to low savings rates, firms have also begun to escalate contribution rates automatically for employees from an initial (low) contribution rate to a higher terminal rate. As of 2012, roughly 70% of all firms with automatic enrollment also engage in automatic escalation: an increase from 30% in

2005. Generally, firms with automatic escalation begin enrolling employees with a 3% contribution rate that increases at a rate of 1% per year to approximately 10% (according to Vanguard, 2013). Strong empirical evidence supports use of default options to increase plan participant contribution rates, and employees of firms with automatic enrollment and automatic escalation contribute significantly more to their DC plans.

Figure 2 illustrates two different savings rates patterns. The SCF contribution rates are inferred from available Survey of Consumer Finances data for the median household. This savings profile can be viewed as an indication of historical savings patterns. The Vanguard contribution rates represent actual contribution rates from employees that currently participate in DC retirement plans with automatic enrollment. Clearly, the contribution rates for retirement plan participants are much higher than historical contribution rates, and these rates are in our view a more appropriate forward-looking assumption for participants in DC plans.

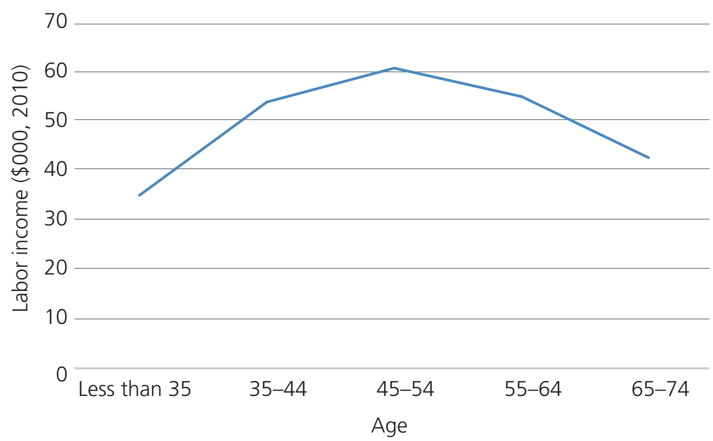
FIGURE 2: DC PLAN SAVINGS TRENDS



Source: PIMCO, Bureau of Labor Statistics, Survey of Consumer Finance (2010), Vanguard (2013).

The timing of DC investments is also very important. Historically, individual labor income has not followed a flat, or even a monotone, trajectory over the lifecycle. The income process we assume for our overall framework is shown in Figure 3.

FIGURE 3: LIFECYCLE MEDIAN LABOR INCOME



Source: Survey of Consumer Finances (2010)

Labor earnings typically are low when individuals begin their working careers and then increase steadily (though at slowing rates) until wages peak in employees' mid-fifties and decline near retirement. The *interaction* of earnings levels with contribution rates determines the timing and flow of assets into the DC plan: 35-year-olds contribute roughly $2,100 per year ($35,000 per year income with a 6% savings rate), while we estimate that 45- to 64-year-olds contribute 2.6 times this amount due to a combination of higher income and higher contribution rates. Note that the slight decrease in incomes for the 55- to 64-year-olds is offset by higher retirement contribution rates. Importantly, it is the timing and flow of assets into the plan that will directly influence the relative importance of retirement assets to the household. Thus, the shape of the household's income profile is a key input into glide path construction. The hump-shaped profile in income and contribution rates suggests that the time period savers in the household are 45 to 55 years has a tremendous impact on their overall DC plan wealth: Not only are they earning the most during these years, but their DC contribution rates are the highest as well.

The modeling framework

In this section we describe the properties of optimal DC glide paths at a high level. The purpose of our analysis is two-fold:

1. Illustrate the importance of specifying retirement objectives in terms of retirement income instead of retirement wealth, and

2. Emphasize the impact that Social Security and housing wealth have on the design of optimal retirement glide paths.

To keep this glide-path analysis concise, we include just three representative financial assets: equities, nominal bonds and long duration real bonds. The simulation procedure and return assumptions we use to generate asset returns, inflation and nominal and real yield curves over the household lifecycle are described in more detail in Appendix 1.

Household preferences over different levels of retirement income are captured by a simple utility function of the constant relative risk aversion (CRRA) form:

$$U(w) = \frac{w^{1-\rho}}{1-\rho}$$

These household preferences are common usage in analytical literature, and they summarize risk preferences by a single parameter, ρ. This specification is especially attractive given the retirement industry's focus on replacement rates: the ratio of income in retirement to income while working. With these preferences, an individual's feelings toward proportional risks are invariant to the level of wealth or income (justifying a focus on replacement rates alone, regardless of the level of labor income). In our primary parameterizations, we use a risk aversion of four, but include sensitivity analysis in subsequent sections. There is a tremendous library of empirical work devoted to calibrating risk aversion preferences. Most reasonable values in the savings and retirement context range from one to five, though the dispersion is quite wide (Kocherlakota [1996], Chetty [2003], Schecter [2007] et al.).

A simple base case: glide paths and human capital

We first consider the optimal asset allocation in the base case for glide-path analysis. First, we assume that households only care about their total wealth at retirement, but have no savings outside their retirement account or access to Social Security benefits. Their only source of retirement income is their funds in their DC plan.

Figure 4 shows the optimal asset allocation for this simplified base case, which features the conventional downward-sloping glide path. This is a universal feature of optimal glide paths, when future labor income is incorporated into the analysis of retirement savings.[2] Indeed, in the retirement industry, glide paths almost universally "de-risk" as the individual nears retirement for this reason. The present value of future labor income behaves like a relatively low risk (in terms of volatility and correlation with risky investments) real asset that amortizes as one nears retirement.[3] Intuitively, future labor income represents a large fixed-income style allocation. This serves to move the initial optimal allocation further into equities when compared to the case where future labor income/human capital is not included in the analysis. We provide a simple derivation of this fact in Appendix 2. Note also that the optimal equity share falls essentially all the way to zero: This reflects the explicit focus on wealth only at the instant of retirement while neglecting the value of any wealth after one retires.

FIGURE 4: SIMPLE BASE CASE: MAXIMIZE WEALTH, ONLY DC PLAN, WITH FUTURE INCOME

Age	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Bonds	0%	0%	28%	48%	65%	76%	88%	98%
Stocks	100%	100%	72%	52%	35%	24%	12%	2%

Source: PIMCO

The "right" retirement objective: from retirement wealth to retirement income

An individual's total wealth at retirement may seem like the obvious measure of a glide path: People generally prefer higher wealth to lower wealth. The reality for retirement, however, is more subtle and complex. The household's retirement consumption takes place over many years (20, 30 or more, given current life expectancies) rather than at a single point in time. In short, individuals implicitly face a *forward real liability*. As a result, changes in *real interest rates* are a critical determinant of financeable real retirement consumption given any realized level of wealth at age 65. There is no "single" wealth number that people should target: At very low real interest rates, substantially more initial investment is required to finance a given real annuity in retirement, particularly for longer life expectancies, as shown in Table 1.

TABLE 1: SENSITIVITY OF REAL ANNUITY PRICES TO TERM AND INTEREST RATES

Cost of a $10,000 real annuity			
	Real interest rate		
Term	0%	1%	2%
10 Years	100,000	94,713	89,826
20 Years	200,000	180,456	163,514
30 Years	300,000	258,077	223,965

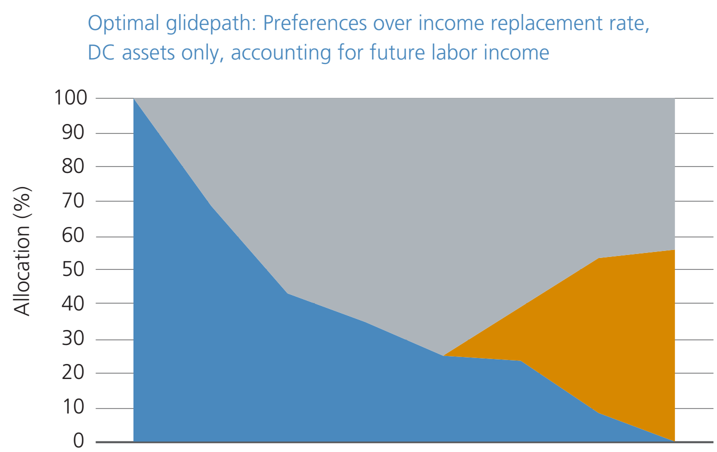
Source: PIMCO

For a 30-year retirement horizon, an increase in the real rate from 0% to 2% per year effectively increases horizon purchasing power by 31% for any given level of wealth. Given the sensitivity of attainable retirement income to the real interest rate, it is more appropriate to evaluate the risk and return of glide- path allocations in terms of the real lifetime income stream that the individual can achieve at retirement.

We next evaluate glide paths in terms of the amount of *real income* that the household can finance by purchasing an actuarially fair immediate real lifetime annuity.[4] The sensitivity of the attainable retirement income to real interest rates clearly suggests that there is a place for long duration real assets within the DC portfolio. To illustrate this, we add long duration real assets (in this case, Treasury Inflation-Protected Securities or TIPS) to the investment universe.

The change in retirement objectives from retirement wealth to retirement income has a significant impact on the optimal glide path, as shown in Figure 5. Long duration real assets are prevalent, particularly in the heart of the glide path, to hedge the real rate risk.

FIGURE 5: FROM RETIREMENT WEALTH TO RETIREMENT INCOME: MAXIMIZE REPLACEMENT RATE, ONLY DC PLAN, WITH FUTURE INCOME



Age	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Long TIPS	0%	32%	57%	65%	75%	61%	47%	44%
Bonds	0%	0%	0%	0%	0%	16%	45%	56%
Stocks	100%	68%	43%	35%	25%	23%	9%	0%

Source: PIMCO

In practice, the ideal glide path would gradually reduce the duration of the allocation to long TIPS (or other long duration real rate hedging assets) in the portfolio to maintain the hedge, rather than coarsely move into lower duration nominal bonds, as in this stylized example. A focus on retirement *income* rather than wealth changes the attractiveness of assets within the glide path.

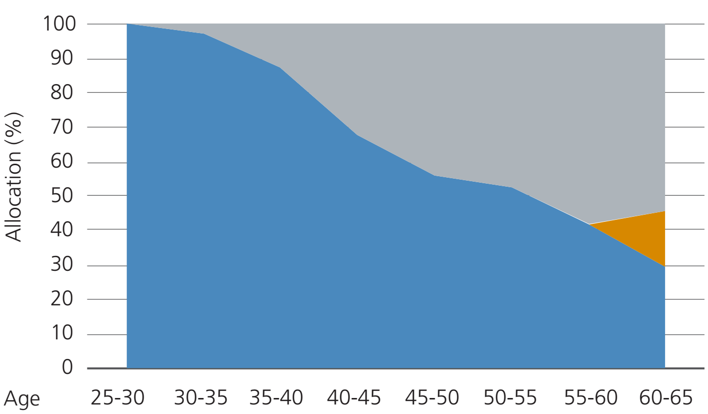
Incorporating the full household balance sheet: Social Security and housing wealth

Thus far, we have focused solely on DC assets and their contribution to retirement income. However, for the vast majority of households, the retirement account is only a small fraction of current assets.

In general, non-DC assets are dominated by accumulated equity in the home. Furthermore, retirement income for most households in the U.S. is dominated by Social Security benefits. These benefits are an immediate real lifetime annuity. The relative riskiness of future income should be assessed in the context of all of the household's assets, and an assessment of the financeable real income stream should include an individual's accrued Social Security benefits. As sources of retirement income and consumption, Social Security and housing wealth have a profound impact on the optimal glide-path allocations.

FIGURE 6: INCORPORATING NON-DC WEALTH AND SOCIAL SECURITY: MAXIMIZE REPLACEMENT RATE, ALL ASSETS AND BENEFITS, WITH FUTURE INCOME

Optimal glidepath: Preferences over income replacement rate, all assets including Social Security benefits, accounting for future labor income



Age	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Long TIPS	0%	3%	13%	32%	44%	47%	58%	54%
Bonds	0%	0%	0%	0%	0%	0%	0%	16%
Stocks	100%	97%	87%	68%	56%	53%	42%	29%

Source: PIMCO

We continue to assume that individuals annuitize all of their non-retirement assets at retirement. But the total wealth now includes housing wealth with a 50% haircut as mentioned earlier. Further, we calculate an individual's Social Security benefits given the assumed income trajectory, current Social Security benefit formulae and simulated inflation rates and add these benefits to the payment from the real annuity purchased at retirement. The optimal glide path incorporating non-DC assets and Social Security benefits is shown in Figure 6.

The equity allocation is now higher throughout the glide path, particularly near retirement. Social Security benefits are equivalent to a guaranteed real retirement annuity that is indexed to household income. As a result, the Social Security "asset" provides households with a limited, perfect hedge against fluctuations in real interest rates and a minimum floor on consumption in the event of substantial portfolio losses. This reduces the desire to hedge against real interest rate risk by investing in long duration fixed income assets. Similarly, other non-DC assets such as the primary residence may act as a hedge against movements in the real interest rate, especially over longer periods of time. Given the magnitude of non-DC assets in Social Security and housing, incorporating them into the analysis affords the DC participant the opportunity to take more risk, hedges against severe consumption loss during retirement and reduces the optimal allocation to long duration fixed income. As before, we also see a small allocation to nominal bonds at the end of the glide path: Given the asset assumptions, this allocation serves to shorten the duration of the assets to match the duration of the terminal liability.

A summary of the changes to the optimal glide path discussed above is provided in Table 2.

TABLE 2: IMPACT OF ASSUMPTIONS OF OPTIMAL GLIDE PATH

Progression to optimal glidepath				
Asset universe	**Naive case**	**Add labor income**	**From retirement wealth to retirement income**	**Incorporating non-DC wealth and Social Security**
Objective	Retirement wealth	Retirement wealth	Replacement rate	Replacement rate
Asset universe				
Stocks	Yes	Yes	Yes	Yes
Bonds	Yes	Yes	Yes	Yes
Long TIPS	No	No	Yes	Yes
Model features				
DC wealth	Yes	Yes	Yes	Yes
Future labor income (human capital)	No	Yes	Yes	Yes
Social Security	No	No	No	Yes
Non-DC wealth	No	No	No	Yes
Summary	Optimal asset allocation is constant over the life-cycle	Optimal asset allocation has a downward sloping profile	Real rate sensitivity of retirement liabilities is very important as Long TIPS dominate fixed income allocation	Allocation to equities increases significantly; Long TIPS dominate fixed income allocation
Starting/ending % in stocks	10% - 20%	100% - 2%	100% - 0%	100% - 30%
Starting/ending % in bonds	80% - 90%	0% - 98%	0% - 55%	0% - 15%
Starting/ending % in long TIPS	n.a.	n.a.	0% - 45%	0% - 55%

Source: PIMCO

Sensitivity analysis for the optimal glide path

Many of the parameters for an optimal glide path – such as the risk aversion of plan participants – simply cannot be known with certainty and may vary significantly across the set of plan participants. Sensitivity analyses around the full balance sheet optimization (including Social Security benefits) described in the preceding section can incorporate the potential variations of these parameters in the optimal glide-path allocation. As with any modeling exercise, changes in the inputs and parameters will change the optimal portfolio.

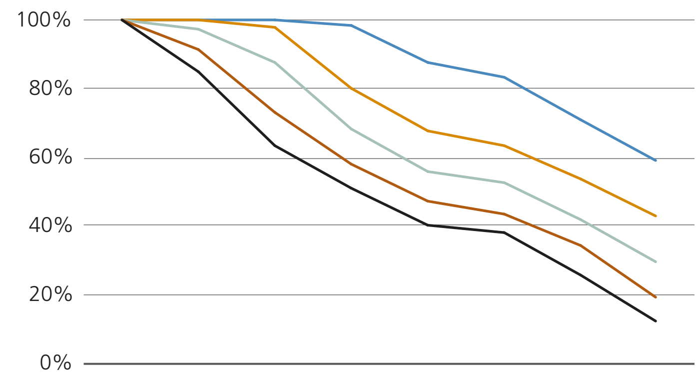
We explore the effects of changes in the following parameters:

- The risk aversion level, ρ
- The equity risk premium and expected equity returns
- Equity volatility
- Starting household wealth (outside the DC plan)
- Wage levels
- Household life expectancy

Taking the first four parameters together, a higher equity risk premium would increase the reward for taking incremental risk and the optimal allocation to equities would be higher relative to long duration. A similar effect would accompany assumptions of higher equity risk premiums and or lower equity volatility. An increase in non-DC wealth reduces the importance of DC assets in the overall balance sheet, and tends to increase the optimal allocation to equities inside the DC plan (see Figure 7).

FIGURE 7: SENSITIVITY ANALYSIS: RISK AVERSION AND RELATED PARAMETERS

Optimal equity allocations with differing coefficients of relative risk aversion (Rho)



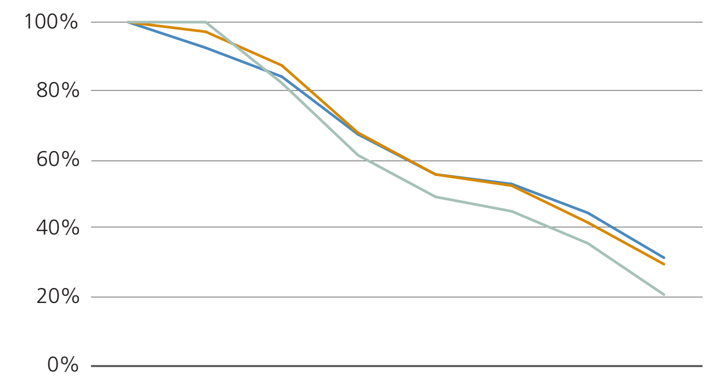
Age	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Rho = 2	100%	100%	100%	98%	88%	83%	71%	59%
Rho = 3	100%	100%	98%	80%	68%	63%	54%	43%
Rho = 4 (Base case)	100%	97%	87%	68%	56%	53%	42%	29%
Rho = 5	100%	91%	73%	58%	47%	44%	34%	19%
Rho = 6	100%	85%	63%	51%	40%	38%	25%	12%

Source: PIMCO

If we change the relative magnitude of future labor income or the dispersion of initial invested assets, either at the outset or over the lifecycle, the optimal glide path itself will change. In Figure 8, we compare our baseline glide path to one with future wages that are higher (by 100%) and lower (by 30%) than our base case. With higher wages, the present value of labor income, and thus of future DC contributions, is higher, which increases the value in equities, at least at the outset of the glide path. However, this effect is relatively small: We only see higher equity allocations for roughly the first 10 years of the glide path. Instead, we have another effect, because Social Security benefits are progressive. With these levels of wages, Social Security benefits alone constitute a 44.4%, 39.7%, and 25.3% replacement rate for the low, medium and high wage profiles, respectively. As Social Security replacement rates decline, the individual has a greater need to hedge against future retirement consumption, which increases the appeal of TIPS and modestly decreases the appeal of equities.

FIGURE 8: SENSITIVITY ANALYSIS: WAGES

Optimal equity allocations with differing wage profiles



Age	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Lower wage	100%	93%	84%	67%	56%	53%	45%	31%
Original wage	100%	97%	87%	68%	56%	53%	42%	29%
Higher wage	100%	100%	82%	61%	49%	45%	36%	21%

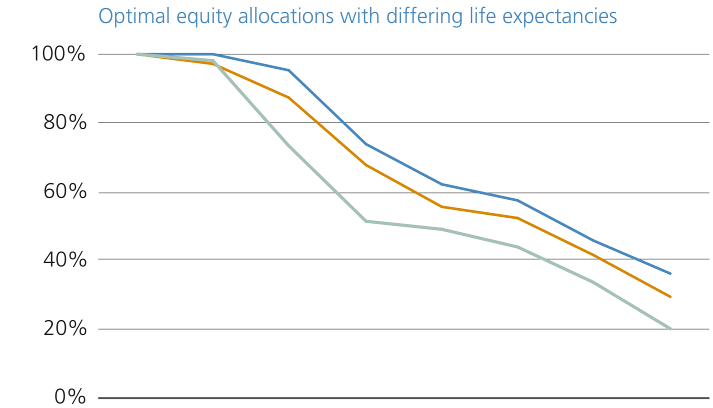
Source: PIMCO

Perhaps the most interesting glide-path changes occur when adjusting life expectancy. When an individual lives longer, the real annuity he or she purchased at retirement becomes more expensive, suggesting an increase in the equities allocation (to provide the additional returns needed). However, as we saw in Table 1, the *duration* of the real annuity increases as well, which will have the second order effect of increasing the appeal of long duration TIPS as a hedge against this risk.

In Figure 9, we calculate the optimal glide paths using the baseline life expectancy (for a 25-year-old male in 2014, life expectancy for when he reaches age 65 is 19.7 years), a 10-year-shorter life expectancy, and a 10-year-longer life expectancy. The appeal of equities increases with life expectancy as the individual needs higher balances at retirement to finance any given level of retirement consumption over that longer post-retirement timeframe. Notice that the first-order effect of needing additional wealth at retirement (favoring equities) dominates the second-order effect of increasing the duration of retirement liabilities (which would make long TIPS more attractive).

Table 3 summarizes the changes in glide paths that are associated with changes in these modeling assumptions. An increase in the risk aversion leads to uniformly lower equity allocations along the entire glide path. This is intuitive: Equities are the highest-risk assets in our investment universe, and they naturally will become less attractive for individuals who cannot tolerate higher risks. The optimal glide-path allocations for risk aversion parameters from two through six are shown in Figure 7. The more risk-averse households now prefer portfolios that further limit the volatility in their retirement income. Similarly, if the risk aversion level was reduced from our assumed level of four, the optimal allocation to equities would be higher.

FIGURE 9: SENSITIVITY ANALYSIS: LIFE EXPECTANCY



Age	1	2	3	4	5	6	7	8
Longer life expectancy	100%	100%	95%	74%	62%	58%	46%	36%
Original life expectancy	100%	97%	87%	68%	56%	53%	42%	29%
Shorter life expectancy	100%	98%	74%	52%	49%	44%	34%	20%

Source: PIMCO

TABLE 3: EFFECTS OF MODELING PARAMETERS ON OPTIMAL GLIDE PATH

Effects of framework and parameters on optimal Glide Path		
Variable	**Effect on equity allocation**	**Effect on long duration allocation**
Increase in risk aversion	Decrease the allocation to equities	Increases the allocation to Long Duration bonds
Increase in expected equity returns	Increase the allocation to equities	
Higher life expectancy	Increases the allocation to equities	Increases the allocation to Long Duration bonds
Increase in initial non DC wealth	Increase the allocation to equities	
Higher wages	Increase initial equity allocation and lowers terminal allocation.	Higher wages increase Long Duration Bond allocations (to compensate for progressivity of Social Security)

Source: PIMCO

Conclusion

Saving for retirement is one of the most important decisions that individuals make during their lifetimes. Often in the asset management industry, retirement account asset allocations (glide paths) are constructed without accounting for several of the most important components of the household balance sheet. Incorporating a household's full asset allocation and projected future retirement liabilities is critical to fully understand the risks embedded within any glide path. Downward-sloping equity allocations are an immediate consequence of incorporating future labor earnings (and retirement savings) into the model. As the present value of these earnings increases, either through increased wages or declining real interest rates, the optimal allocation to equities will increase. Benchmarking the model's objective function to retirement consumption introduces sensitivity to real interest rates that in turn increases the appeal of long duration real assets (such as TIPS) throughout the glide path. However, households typically accumulate substantial real rate hedges outside of their retirement account: through accrued Social Security benefits, which are explicitly indexed to inflation, or through home ownership, which is commonly seen as a hedge against inflation.

Taken together, these suggest a glide-path allocation mixed between equities and real-rate-sensitive assets, such as long TIPS. From there, the particular parameterization of the simulation determines the relative trade-offs: More risk-averse households will prefer less equities; higher initial DC wealth makes risk in the DC account relatively more consequential, which pushes the optimal glide path out of equities; longer life expectancy increases the amount of assets required at retirement to finance any given level of consumption, which increases the appeal of risky assets.

We believe that glide-path construction needs to include an evaluation of future labor income, needs to have performance measured against retirement liabilities (and thus be benchmarked against the real interest rate), and must include Social Security benefits and housing wealth (which may free the household to invest more aggressively and encourage diversification).

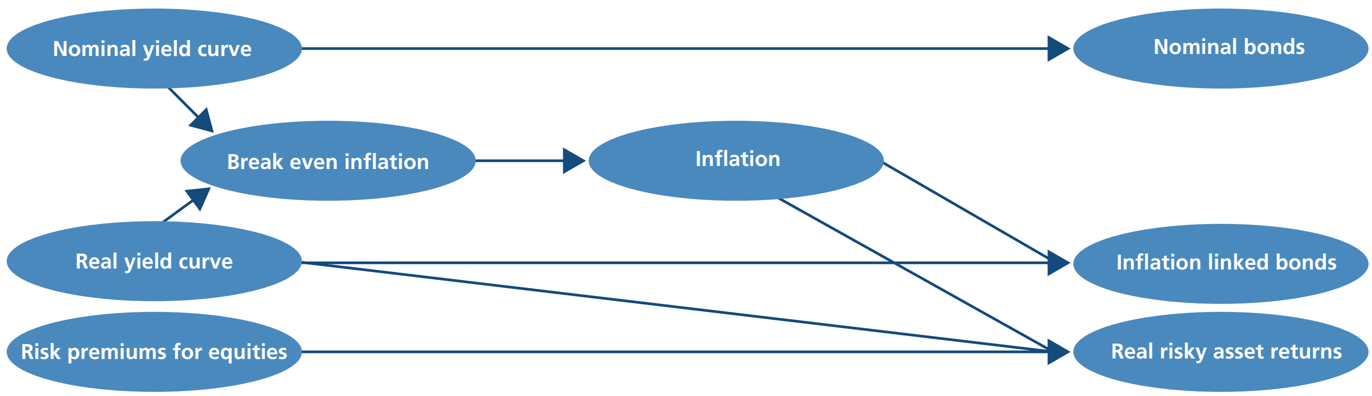
In a theory-based analytical tool, several key mechanisms and parameters can be modeled and combined into a robust framework to assess glide-path design. In practice, additional asset classes can be included in the implementation of the glide path. For example, long nominal bonds may be attractive relative to long duration inflation-linked bonds or commodities, real estate investment trusts, global equities and bonds, and alternative asset classes may offer some diversification relative to equities or additional real rate protection. As a result, the final implementation of an optimal glide path will often incorporate additional asset classes that can increase expected returns or achieve additional long-term diversification benefits.

THE PIMCO GLIDE PATH DIFFERENCE

PIMCO is committed to offering the best possible solution to help DC plan participants achieve successful retirement outcomes. In order to do that, we carefully assess our glide path each year. This review allows us to reevaluate our prior assumptions, contemplate any adjustments based on our Secular Forum conclusions and incorporate analytical enhancements in our modeling. As part of the 2014 review, our Analytics team enhanced a number of components of the quantitative analysis that underpins the glide path. Specifically, we incorporated and quantified human capital and liability matching considerations more explicitly (also described in the *In-Depth article*, **"Using a Real Liability to Assess Retirement Readiness and Inform Investment Decisions"**). Finally, empirical data related to the evolution of participants' overall household wealth were used to enrich the model. The results of our analysis are compelling, and they are in direct alignment with the key tenets our glide path has always been based upon. We are excited to share this thought leadership with the market.

Appendix 1: The Simulation Process

OVERVIEW OF SIMULATION ENGINE



Source: PIMCO

The simulation of nominal yield curves follows a three-factor model Cox-Ingersoll-Ross process, while real rates follow and we use a three-factor (normal) Vasicek model. Realized inflation is tied to the dynamics of break-even inflation rates implied by the joint simulation of real and nominal rates. Both real (inflation-linked) and nominal bond returns at all relevant maturities are directly determined by the simulated yield curves. When real rates are low, real income from a given mix of assets (especially fixed income) is low.

Nominal Rates Process

A reduced form three-factor log-normal Cox-Ingersoll Ross (CIR) model governs the dynamics for the term structure of nominal rates,

$$dy_2 = \left(\Theta_{2,y} - y_2\right)dt + \sigma_{2,y}\sqrt{y_2}dz_{2,y}$$
$$dy_{10} = \left(\Theta_{10,y} - y_{10}\right)dt + \sigma_{10,y}\sqrt{y_{10}}dz_{10,y}$$
$$dy_{30} = \left(\Theta_{30,y} - y_{30}\right)dt + \sigma_{30,y}\sqrt{y_{30}}dz_{30,y}$$

Real Yields

A reduced form three-factor Vasicek model governs the dynamics for the term structure of real rates,

$$dr_2 = \left(\Theta_{2,r} - r_2\right)dt + \sigma_{2,r}r_2dz_{2,r}$$
$$dr_{10} = \left(\Theta_{10,r} - r_{10}\right)dt + \sigma_{10,r}r_{10}dz_{10,r}$$
$$dr_{30} = \left(\Theta_{30,r} - r_{30}\right)dt + \sigma_{30,r}r_{30}dz_{30,r}$$

where $dz_{2,r}$, $dz_{10,r}$, $dz_{30,r}$ are jointly normally distributed. As with nominal rates, the real term structure is fitted using a four-factor Nelson-Siegel model and is used to calculate returns for all real assets, where $dz_{2,y}$, $dz_{10,y}$, $dz_{30,r}$ are jointly normally distributed and y_2, y_{10}, y_{30} correspond to the two-year, 10-year and 30-year yields, which allows for straightforward calibration of term structure volatilities, correlations and term premiums. Given the simulated path for two-, 10- and 30-year yields, the full term structure of nominal yields is fitted on each path of the simulation to a four-factor Nelson-Siegel model. The yield process characterizes the associated process for nominal bond returns at all maturities from one year to 30 years out.

Inflation and break-even inflation

The process for the term structure of break-even inflation is defined by the difference between the term structure of nominal and real rates; we define short-term break-even inflation as the difference between the simulated one-year nominal rate and the simulated one-year real rate, so

$$\pi_1 = y_1 - r_1$$

Realized inflation in a given year is then simulated as break-even inflation plus a stochastic term

$$\pi_{infl} = \pi_1 + \sigma_{infl} v_{infl,t}$$

In this way, the dynamic process for break-even inflation is implicitly defined by the dynamics of the difference between nominal rates and real rates, and the actual realized inflation process is consistent with the short-term break-even inflation rate at each point in the simulation.

Equity and real estate returns

Long-horizon risky asset returns use a parsimonious specification of returns that build the expected return as risk-free rate plus a risk premium (see comments in previous description of model). As an example, simulated real equity returns are based on the following specification

$$x_{eq} = r + \lambda_{eq} + \sigma_{eq} dz_{eq}$$

where λ_{eq} is the (term structure of) equity risk premium. The same basic model specification is used for equity assets. The risk premium, volatilities and correlations vary across all risky assets according to current valuations and historical return patterns.

The process for house prices (when we simulate primary residence values) is linked to inflation over time such that on average, real home prices are expected to stay constant in the long run and the *expected real price* return from ownership in the primary residence is zero. This is broadly consistent with long-term evidence across countries, though short-to-medium term deviations from this level are not uncommon, particularly in recent decades.

$$\pi_{hp,realized} = \pi_{infl} + \mu_{hp} + \sigma_{hp} v_{hp,t}$$

Benchmark parameterization

The table below summarizes our parameter assumptions for the simulation of asset returns, yield curves and inflation.

Variable	Parameter	Value	Parameter	Value
Nominal rates				
2 year nominal key rate	$\Theta_{2,y}$	3.50%	$\sigma_{2,y}$	150 bps vol.
10 year nominal key rate	$\Theta_{10,y}$	4.00%	$\sigma_{10,y}$	100 bps vol.
30 year nominal key rate	$\Theta_{30,y}$	4.25%	$\sigma_{30,r}$	85 bps vol.
Real rates				
2 year real key rate	$\Theta_{2,r}$	1.00%	$\sigma_{2,r}$	100 bps vol.
10 year real key rate	$\Theta_{10,r}$	1.25%	$\sigma_{10,r}$	85 bps vol.
30 year real key rate	$\Theta_{30,r}$	1.50%	$\sigma_{30,r}$	65 bps vol.
Inflation				
Realized inflation	$E[\pi_1]$	2.25%	σ_{infl}	60 bps
Equity				
Equity risk premium (arithmetic)	λ_{eq}	4.0%	σ_{eq}	20% vol.
Equity risk premium (geometric)	$\lambda_{eq} - 0.5\sigma^2_{eq}$	2.0%		
House Prices				
	μ_{hp}	0.0%	σ_{hp}	5% vol.

Source: PIMCO

Overall, the benchmark parameterization is calibrated to produce realistic long-term dynamics for bond returns, equity returns, nominal and real rates, and inflation. The volatility levels are calibrated to be in line with historical experience. We assume a long-term average inflation rate of 2.25%. Long-term 10- and 30-year nominal yields converge to an average of 4% and 4.25% respectively along with a 3.5% average two-year nominal rate. As such we build in a term premium in the yield curve of 50 bps at the 10-year maturity and 75 bps for 30-year rates. The real yields converge to 1%, 1.25% and

1.5% for the two-year, 10-year and 30-year maturities. Consistent with empirical data, real yields are assumed to be somewhat less volatile than nominal yields in our simulation.

The equity risk premium is assumed to be 4% in arithmetic terms and 2% in geometric terms, which is considerably lower than the long-term historical averages of 7.5% and 5.5%, respectively. The moderate equity risk premium is consistent with current pricing in equity markets and also consistent with a gradual reduction in the ex ante equity risk premium that investors have required in global equity markets.

Appendix 2: Deriving a Downward-Sloping Glide Path

The Model

In this model, we extend the continuous-time Merton problem to include a stream of non-asset-based income and immediately recover a downward-sloping glide path as the optimal allocation. The individual selects a sequence of allocations to risky assets out of their financial wealth, $\{\phi_t\}_{t=0}^{\infty}$, each period to solve:

$$\max_{\{\phi_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\beta s} U(c_s) ds \qquad (1)$$

where each period, the individual has a present value of labor income equal to I_t, and a total wealth equal to W_t (so financial wealth equals $W_t - I_t$). If the income received each period is equal to i (and constant through time), we have[5]:

$I_t = \int_t^{\infty} e^{-rs} i\, ds$, which implies that dI_t is not a function of ϕ at any point in time. Note that stochastic (or uncertain income) would introduce additional terms (below) that would further complicate the expressions, but would not materially affect the results for realistic income-asset correlations or unemployment hazard rates. We assume the evolution of the risk-free and risky assets are given by:

$$dB_t = rB_t dt \qquad (2)$$

$$dS_t = \mu S_t dt + \sigma dZ_t \qquad (3)$$

where Z_t is a standard Brownian motion, r is the risk-free rate, μ is the expected return on the risky asset, and σ is the volatility of the risky asset. This implies that total wealth follows the process:

$$dW_t = \left((1-\phi_t)r + \phi_t \mu\right)\gamma_t W_t - c_t)dt + \phi_t \gamma_t W_t \sigma dZ_t dt \qquad (4)$$

where γ_t is the financial asset's share of wealth $(\gamma_t = \frac{W_t - I_t}{W_t})$. We can rewrite this problem in terms of the Jacobi-Bellman equation as:

$$V(t, W_t, I_t) = \max_{\phi_t}[U(c_t) + E(dV)] \qquad (5)$$

Combining equations (4) and (5) and applying Ito's lemma, we have:

$$dV(t, W_t, I_t) = V_t + V_w\left[\left((1-\phi_t)r + \phi_t \mu\right)\gamma_t W_t - c_t)\right] + \frac{1}{2}V_{ww} \left[\phi_t{}^2 \sigma^2 (\gamma_t W_t)^2\right] + V_I(dI_t) \qquad (6)$$

Using equation (6), we can solve for the first order conditions for investment in the risky asset:

$$-V_w(\mu - r) = V_{ww}\gamma_t W_t \phi_t \sigma^2, \text{ or} \qquad (7)$$

$$\phi_t{}^* = \frac{-V_w}{V_{ww}W_t}\frac{(\mu - r)}{\sigma^2}\frac{1}{\gamma_t}$$

At the same time, we have a first order condition for consumption (the envelope condition) given by:

$U'(c_t) = V_w$, which for CRRA preferences $(U(c) = c^{1-\theta}/(1-\theta))$ reduces equation (8) to the unconstrained form with an additional term, $\frac{1}{\gamma_t}$, the inverse of the financial asset's share of total wealth.

$$\phi_t{}^* = \frac{(\mu - r)}{\theta \sigma^2}\frac{1}{\gamma_t} \qquad (8)$$

So, in addition to the typical comparative statics, i.e., that the optimal allocation to the risky assets increases with expected excess return, and decreases with both risk aversion and the volatility of the risky asset, we have that the optimal allocation to the risky asset increases when the share of financial wealth decreases. In the context of individuals, the present value of their labor income when young (I_t) will dwarf their initial financial assets, suggesting a more aggressive allocation early in the lifecycle that gradually declines until the present value of non-asset income reaches zero, after which the optimal allocation would remain stable and equal to the optimal allocation from the problem without any labor income.[6]

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[1] Figures represent median asset accumulation for the indicated quantile. Initial differences in median by quantile are adjusted by the evolution of asset balances by age from the 2010 Survey of Consumer Finances. Realized asset accumulation behavior may be very different. Graphically, Social Security represents the accumulated benefits, rather than their expected present value, though this has minimal impact on the key results.

[2] If one were to ignore labor income and instead simply manage a given amount of initial wealth until retirement, the optimal allocation is a static, constant allocation over time. Regardless of the time to retirement, people should choose the same asset allocation and there would be no need for "glide paths." This result was first demonstrated in Merton (1969).

[3] This also implies that in an era of prolonged low rates as forecast in PIMCO's New Neutral hypothesis, all else equal, the present value of future labor income will increase, which will increase the initial allocation to equities and steepen the glide path.

[4] For this calculation, we use mortality tables and forward life expectancies from the Social Security Administration (2013), assuming a participant age of 25 years in 2014. We normalize the real annuity by the household's terminal real labor income to calculate a lifetime real income replacement rate.

[5] Adding randomness, including correlated randomness, to the income process introduces another term to equations (2) and (7), though it does not substantively affect the comparative statics highlighted in this section.

[6] Obviously, the optimal allocation will not be constant with preferences other than CRRA. Note also that in this model, all non-asset-based income, such as Social Security and pension benefits, would also be included in I_t and would serve to permanently bound the optimal equity allocation above the unconstrained optimum.

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